Motivations and Physical Aims of Algebraic QFT

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Abstract

We present illustrations which show the usefulness of algebraic QFT. In particular in low-dimensional QFT, when Lagrangian quantization does not exist or is useless (e.g. in chiral conformal theories), the algebraic method is beginning to reveal its strength.

This work is dedicated to the memory of Claude Itzykson

1 History and Motivation

One characteristic feature which distinguishes algebraic QFT from the various quantization approaches to relativistic quantum physics based on classical Lagrangians (canonical, path-integral), is its emphasis on locality and its insistence in separating local from global properties. The former reside in the algebraic structure of local observables, whereas the latter usually enter through states and the suitably constructed representation spaces of local observables.

The idea that the "Global" constitutes itself from the "Local" is of course the heart-piece of classical electromagnetisms as formulated by Faraday, Maxwell and Einstein.

Algebraic QFT is more faithful to physical principles than to particular formalisms. As such it has more in common with the Kramers-Kronig dispersion relations of the 50's as a test of Einstein causality, than with the post Feynman developments of functional formalisms. Its conceptual and mathematical arsenal is however significantly richer than the QFT underlying the derivation of the Kramers- Kronig dispersion relations.

When physicists first analyzed the new concepts of quantum theory, they payed little attention to local structure since their main aim was to understand the new mechanics. For example in von Neumann's early general account of the mathematical framework of quantum physics in terms of observables and their measurement, locality and causality did not enter at all. Even in the subsequent refinements of Wick, Wightman and Wigner [1] concerning limitations on the superposition principle, locality was not used.

The idea that, by combining the superselection principle with relativistic locality, one may obtain a powerful framework in which global superselected charge sectors including their (asymptotic) particle structure together with the statistics could emerge from the more fundamental "nets" of local observables, can be traced back to the work of Rudolf Haag in the late 50's [2].

More than a decade before, E.P. Wigner [3] had successfully demonstrated that it is possible to investigate fundamental issues of relativistic quantum physics without any recourse to quantization. Here, we are of course referring to his famous representation theory for the Poincaré -group which ended the long-lasting (but somehow academic) discussion of relativistic linear field equations.

By the end of the 50's, there also existed an "axiomatic" framework [4] (the "Wightman framework") for point-like covariant fields, which, if enriched by the LSZ asymptotics [5], gave a unified viewpoint linking the 1929 Heisenberg-Pauli canonical commutation approach [1] [6] with the 1939 Wigner representation theory under one roof. [2]

[1]We will, whenever possible, refer to textbooks or reviews where the content of historical papers is presented from a viewpoint similar to ours.

[2]Behind these elegant schemes which were based on a few principles was of course the sweat and tears of the pioneers of renormalized perturbation theory.

The beginning of what is nowadays called algebraic QFT is usually identified with the 1964 paper of Haag and Kastler [7], although there were prior important developments notably by Borchers and Araki [7]. Borchers showed among other things, that the scattering matrix is an invariant shared by a whole local class of fields (the "Borchers equivalenc class"), and Araki demonstrated that the local observable algebras of algebraic QFT have no minimal projectors (i.e. they allow no maximal measurements), even for the case of free fields. In contemporary mathematical terms they are type III_1 hyperfinite von Neumann factors, a fact which (as will be commented on later) has deep physical relations to such diverse looking issues as the existence of antiparticles and the Hawking-Unruh effect. The derivation from the physical principles of local observables of the properties of charge-carrying fields, including their statistics and internal symmetries, was an important result of the early 70's and is nowadays referred to as the DHR theory [7]. Its specific adaption to low-dimensional QFT with $d \leq 2+1$, which leads to braid group statistics, is of a more recent vintage [8].

For reasons of completeness we should add the remark that nontrivial superselection rules (generalized "θ-angles") can also be obtained without local observables in (global) quantum mechanics. However, in that case, strictly speaking, one has to go beyond the Heisenberg-Weyl-Schrödinger theory and consider C^*-algebras with finite degrees of freedom which, like the particle on a circle (the "rotational C^*-algebra") or the rigid top (C^*-SO(3) group algebra), are not simple and hence admit inequivalent irreducible representations. In all those global cases, the superselection rules are *not fundamental* but rather result from a *physical over-idealization*. Take the example of the string-like Aharonov-Bohm solenoid. This is of course the idealization of a finitely extended physical solenoid in Schrödinger-theory. It so happens, that in the singular limit, when the mathematics simplifies, the C^*-algebra becomes the rotational algebra, which is different from the Heisenberg-Weyl algebra. The fact that there is a single C^* algebra with inequivalent irreducible representations is often overlooked in the differential geometric (fibre bundle) framework which tends to interpret different representations as different theories, and not as superselected sectors of just *one* C^*-algebra.

Only in local theories with infinite degrees of freedom i.e. quantum field theories (also in the thermodynamic limit of lattice theories) one may encounter fundamental superselection rules as a natural consequences of the local observable structure (and the various globalizations it naturally leads to). It is more important to stress the differences [3] of local quantum physics (with entirely new concepts) with respect to quantum mechanics than some similarities between them.

The assumptions on which the algebraic method is founded can be separated

[3]To view local quantum physics as just a particular extension of quantum mechanics or a quantized extension of classical field theory is in my view somewhat misleading. It is rather a new realm of physics with novel concepts.

into two groups.

(1) *The local algebraic net properties for the observables.*

They are formalized in terms of a coherent map $\mathcal{A}$ of finite space-time regions O into von Neumann-algebras:

$$\mathcal{A} : O \to \mathcal{A}(O)$$

It is sufficient to define this "net" (i.e. family of von Neumann algebras indexed by space-time regions) for the smallest family of regions stable under Poincaré-transformations which is the family $\mathcal{K}$ of "diamonds" or double cones. It is generated from double cones centered symmetrically around the origin by applying Poincaré transformations. The local algebras $\mathcal{A}(O)$ are operator algebras in an auxiliary Hilbertspace. Among the coherence properties they satisfy is the space-like commutativity (Einstein causality).[4] Often the auxiliary Hilbertspace is chosen to be physical the representation space of the vacuum representation.

The net of von Neumann algebras lends itself to two globalizations: the C^*-algebra of quasilocal observables (the inductive limit for families $\mathcal{K}$ directed towards infinity) $\mathcal{A}_{quasi}$ [7], or the larger universal observable algebra $\mathcal{A}_{uni}$ which corresponds to an algebraic analogue of geometric space-time compactification. space-time [9].

(2) *The global quantum aspects of states*

Since the (globalized) net allows for many inequivalent representations, one is forced to work with a more general concept of states than that of Schrödinger quantum mechanics [7]. In QFT states are linear positive functionals on the observable net which satisfy certain continuity properties with respect to this net. Among the physical state there is the distinguished vacuum state. Behind the vacuum requirement (or the use of the closely related KMS temperature states) is the stability idea [7, 13].

At this point it is helpful to alert the reader against potential misunderstanding of the algebraic approach as some form of "axiomatics" or "doctrine" on local quantum physics. As already in case of von Neumann's concepts of observables, the idea of a net of local observables is nothing more than a useful "ordering principle of the mind" in order to separate a part of reality about which we can safely apply our intuitive locality and causality principles, from useful additional constructions like e.g. charge-carrying fields with weaker and less immediate localization properties. For the latter our intuition is less reliable, since it is based more on experience with formalisms and less on physical principles. Whether one extends observable nets or makes them smaller, does not influence the properties of the complete theory (in as much as in the analogous problem of Heisenberg's

[4]This *locality principle* was, and continues to be (see also section 5 on chiral conformal field theories) the *heart-piece of the algebraic approach* and local quantum physics.

cut between the system and the observer in the process of measurement the exact position of the cut is flexible as long as there is a cut somewhere).

Looking at the dichotomy between observables and states the question "where are the covariant pointlike fields?" will invariably arise. This question is best answered through an analogy with differential geometry. The fields serve as a "coordinatization" of the local algebras. They play the role of a kind of pointlike generators, and very different looking systems of fields may generate the same local algebras. The intrinsic physics resides in the local net and its physical states, but for explicit model calculations it is convenient to use fields. On the other hand the structural properties of the theory are more appropriately analyzed within the net formalism. Certain "coordinates" are however distinguished, e.g. local Noether currents and charge carrying fields with the smallest dimension (the relevant variables in the language of critical phenomena) in their respective superselection class.

The deemphasis on fields in favour of local nets is typical for the algebraic method as compared to quantization approaches. Once a property has been understood within the algebraic setting, it is clear that one is dealing with an intrinsic aspect of the physical system i.e. an aspect which does not depend on the formalism by which the theory has been constructed. Not all properties obtained by applying the various quantization approaches are intrinsic in this sense, as examples will show later on. The notion "intrinsic" which I use throughout this article has an analogous significance as "operational" or "observable" in the early days of quantum mechanics when "Gedankenexperiments" were used for clarification. Here its meaning is a bit more technical. Properties which can be reconstructed by just using the physical correlation functions are called intrinsic and structures which depend on the path of construction and cannot be reconstructed from the observables (example: local gauge invariance, axial anomalies) are extrinsic.

Among the important discoveries, which originated from a quasi-classical or perturbative view about QFT that were successfully incorporated and extended by the algebraic method, are the Goldstone-Nambu mechanism of spontaneous symmetry breaking and Schwinger-Higgs "screening" mechanism in abelian gauge theories. In algebraic QFT these ideas are elevated to the level of structural theorem on equal footing with the TCP and Spin and Statistics Theorem [10, 11, 12].

Unlike the Wightman framework, the algebraic method, as previously mentioned, does not make any a priori assumptions on the form of commutation relations between charge carrying fields. The representation theory (superselection theory) of observable nets is powerful enough to determine those commutation relations (modulo Klein transformations, which are then fixed by invoking locality) [26].

Even perturbation theory, if looked upon in the spirit of algebraic QFT, presents new and interesting problems. Here the recent discovery of the "microlo-

cal spectrum condition" [13], following an idea of Radzikowski, has been instrumental in the re-surging interest in perturbation theory with algebraic methods. It is interesting to note that this local form of spectrum condition was discovered in the quantum field theory in curved space-time, where a good substitute for the (in that case meaningless) global vacuum reference state (in form of a new principle behind the recipe of the "Hadamard condition" [13] or the "adiabatic vacuum" prescription) had been particularly pressing. More comments on these very recent results can be found in section 5.

As far as local gauge theory is concerned, the difficulties which algebraic QFT had in attributing an intrinsic meaning to an elusive "local gauge principle" have deepened. Comments on this problem can be found in many papers on algebraic QFT and there are examples of low dimensional gauge theories which allow for an alternative formulation without local gauge fields and anomalies, thus showing that the local gauge formulation is at best an option, but not a principle [14].

A recent criticism of the "gauge principle" appears in the work of N. Seiberg [15] in supersymmetric gauge theories. However there remains a difference between the Seiberg-Witten interpretation of the asymptotic freedom and the infrared limit regime as phases of the theory versus that of algebraic QFT where e.g. the short distance theory reveals itself as *another* theory which is related to the original one in a very interesting way and generically has more degrees of freedom than the original [16]. This enhancement of global symmetry ($\simeq$ enhancement of degrees of freedom) is unfortunately not typical for local gauge theories. It happens abundantly in two dimensional QFT (example Ising field theory) where the chiral limit has significantly more sectors than its massive parent theories. The phenomenon is related to the emergence of nontrivial dual (order-disorder) charges in the scaling limit whereas in gauge theory it corresponds to short-distance de-confinement [16].

With all these differences in interpretation and formalism it should come as no surprise that algebraic QFT also leads to a different view on the Schwinger-Higgs screening mechanism. First some historical remarks are in order. In an interesting early note [17], Schwinger pointed out the possibility that charged matter interacting through electromagnetism may suffer a charge screening accompanied by the appearance of massive photons. He did not think in terms of "condensates" formed by the matter fields. Since he had no rigorous argument in $d = 3+1$, he invented the $d = 1+1$ "Schwinger model" [17] in order to illustrate the possibility of a "screened phase" which he imagined to occur in $d = 3+1$ theories for strong couplings.

Later Higgs [18] (apparently being unaware of Schwinger's work) studied scalar electrodynamics and found a perturbative regime with a condensate of the matter field and a massive photon. He phrased his findings within the current framework of local gauge theories and thought about the massive photon as originating from a two-step process; first a spontaneous symmetry breaking with a Goldstone-boson and then the photon "swallowing" this boson and be-

coming massive. Lowenstein and Swieca [19] made Schwinger's arguments in the $d = 1 + 1$ model more transparent and showed that there is a chiral symmetry breaking which leads chiral to condensates. Since there is no Goldstone-Nambu mechanism in $d = 1 + 1$, one notes that this model does not allow the aforementioned two-step interpretation.

As already mentioned before, Schwinger's ideas were vindicated in a beautiful structural theorem proved by Swieca [11]. The theorem assumes that the charge-currents are the sources of a Maxwell field, but uses no "local gauge principle" or condensates. For this reason it is very much in the spirit of algebraic QFT, since the latter are less intrinsic concepts. But it seems that the more manipulative but less intrinsic condensate viewpoint became more popular than Schwinger's screening picture.

Algebraic QFT views the would be charge carriers not as point-like covariant (and hence gauge dependent) fields, but rather as semi-inifinitly extended locally gauge invariant objects [20] whose localization regions are space-like cones (in the singular limit: space-like semi-infinite Mandelstam strings). Although they are locally gauge invariant, they carry a global gauge charge.

In the Schwinger-Higgs phase of scalar electrodynamics these locally gauge invariant extended fields condense into the vacuum (and in this way they improve their localization properties by loosing their infrared photon clouds). At this point it would have been nice to illustrate this within the path-integral perturbative setting and the Faddeev Poppov program including the gauge fixing. But I was unable to find an appropriate reference. Apparently the problem of conversion of the path-integral manipulations into the physically clearer language of operator algebras did not enjoy much popularity (it cannot be done by differential-geometric techniques). However the Schwinger model provides a nice rigorous operator illustration of these nonlocal objects in $d = 1 + 1$ [16].

In sections 2 and 3 I will discuss the physical aims of the algebraic method in the context of low dimensional QFT where they appear presently in their clearest form through the braid-group statistics problem. It has been known for some time, that chiral conformal QFT furnishing the simplest analytic illustration of braid group statistics. The exchange algebras carrying this statistics (with the R-matrix structure constants) follow from the aforementioned principles in a natural way [21]. They represent a specialization of a more general and abstract [8] concept of algebraic QFT. In recognition of the fact that the richness of chiral superselection rules and braid-group statistics results from the aforementioned locality and stability principles (and not just from special ad hoc algebras as Kac-Moody algebras (current algebras) etc.) we called one of our contributions [21] "Einstein causality and Artin Braids". However I also should not hide the fact that my attempts to understand certain global properties as e.g. the "modular structure" in the sense of Kac and Wakimoto [49] in the framework of algebraic QFT were less than successful [50]. Since an understanding of these modularity properties as a consequence of the principles (perhaps with the help

of the powerful more general Tomita-Takesaki modular theory) is a physically [5] fascinating challenge (and perhaps besides the braid structure the only physical reason for being interested in chiral theories), I will certainly try a second time. It is encouraging that at least a partial result in this direction has been obtained very recently [51].

The reason behind this analytic simplicity of chiral conformal QFT is that on the right or left light cone a genuine interaction is not possible (a kind of algebraic Huygens principles). The charged chiral conformal fields should be viewed as the "freest" fields which fulfill physically admissible charge-fusion rules together with the uniquely affiliated and measurable plektonic statistics (or more precisely R-matrix commutation relations, since chiral conformal QFT has no Wigner-particle structure). These new fusion laws, unlike those of compact symmetry group representations, are not compatible with the Lagrangian formalism. In fact, these fields strictly speaking are not even Wightman fields since they come equipped with source and range projectors. The problem of whether one can rescue the Lagrangian formalism by combining right and left chiral fields to local fields is in my view of an academic nature, since all the computational power resides in the non-Lagrangian chiral fields. The Lagrangian quantization approach is a useful tool whenever it applies, but certainly not a concept required by physical principles [22].

In $d = 2 + 1$ plektonic theories the charges and their fusion laws become also attributes of particles. Like in $d = 3 + 1$, a given set of superselection rules admit many solutions which are distinguished by coupling constants (which may be thought of as dynamical deformation parameters). There the idea of freeness may be used to select a distinguished field. The best way to implement this idea seems to proceed by an extension of the Wigner representation theory [23] as in section 6. Again it seems that Lagrangians play no useful computational role as soon as plektonic statistics appears.

We finally would like to attempt to incorporate algebraic QFT into the history of ideas on relativistic quantum physics. The most influential method of thinking which has dominated QFT for almost three decades is that of Dirac which is based on geometric intuition, formal mathematical elegance and the power of analogies. The "square root" of the Klein-Gordon operator, the "filling of the Dirac sea" (in order to implement the idea of stability in relativistic quantum physics) and the geometric construction of magnetic monopoles may serve as illustrations of the "Diracian" mode of thinking.

Even if we nowadays see things in a slightly different way, nobody can deny that this approach to fundamental problems has been extremely successful. It appears to me that the discovery of the electro-weak (phenomenological) theory

[5]This structure appears precisely at the place of the "Nelson-Symanzik symmetry" for the Euclidean interchange of space and time, i.e. at one of the most profound discoveries of the 60^{ies}.

has been the last real big physics conquest made in this geometric spirit. All attempts to go beyond this discovery (GUT, strings, supersymmetry and probably also most of the more recent inventions) were not met with success. This failure, which by the end of this century has developed into a profound crisis of particle physics, QFT, and even of adjacent areas, may be the result of an exaggerated unbalanced use of the Diracian approach, rather than an inherent weakness of this method.

On the other hand, there has also been a tradition which emphasizes the importance of physical principles as the condensated form of all our past experience. This mode of thinking always confronts actual findings with these principles, often by the intervention of Gedankenexperiments. It develops its full strength, if this confrontation leads to an antinomy or paradoxon. One finds this mode of thinking particularly in the works of Bohr and Heisenberg, in fact it was essential for the discovery of quantum mechanics by Heisenberg. (Schrödinger was perhaps a bit more on the Diracian side.) Also the later contributions of von Neumann, Jordan, Pauli and Wigner serve as good illustrations. For the purpose of our presentation, let us call this mode of thinking "Bohr-Heisenbergian". A more recent example for this approach is algebraic QFT.

I do not know a more appropriate illustration for the difference between the "Diracian" and "Bohr-Heisenbergian" way of doing local quantum physics than the two different ways in which extended objects (in particular with string-like objects) enter local quantum physics.

The Diracian physicist goes beyond point-like (or compactly localized) objects for reasons of esthetical pleasure related to mathematical consistency and beauty. On the other hand he is able to be somewhat lighthearted about the underlying physics (viz. the sudden transition of the string formulated dual model of strong interactions to alleged semi-classical approximation of quantum gravity of modern string theory). The development of string theory from the dual model was certainly a "Diracian" contribution.

A "Diracian" physicist likes inventions (supersymmetry, superstrings, quantum groups,...) and sometimes even takes them for discoveries.

Quite differently the "Bohr-Heisenbergian" physicist generally shuns away from invention and tends to be less impressed by mathematical beauty unless it arises as a result of compelling conceptual insight. If string-like objects are necessary in order to fully explore a physical principle, he is able to full-heartedly embrace such objects.

This was indeed precisely the way how string-like objects entered algebraic QFT. At the beginning of the 80's Buchholz and Fredenhagen [4, 24] proved a structural theorem (following up some problems raised in Swiecas work [11]) which established the arbitrary thin space-like cone ("semi-infinite string-like") localization of charge-carrying fields as the a priori best possible localization resulting from the physical principles of algebraic QFT (together with the mass gap hypothesis). Later [8, 9] it became clear that in low-dimensional theories this

more general localization really occurs in connection with braid group statistic theories, e.g. in $d = 2 + 1$ braid group statistics exhausts this semi-infinite string-like localization in the sense that any better localization (i.e. compact or point-like) necessarily leads back to permutation group statistics. In other words only charge-carrying operators with this semi-infinite extension are able to carry "plektonic" charges fulfilling the new fusion laws which are not covered by the representation theory of compact (internal symmetry) groups. Such fields do not appear naturally in Lagrangian quantization although attempts have been made to describe such situations via a Chern-Simons extension of the standard quantization formalism.

In in the Bohr-Heisenbergian mode of thinking, the discovery of the connection between localization and statistics is interpreted as an extension of the Einstein-causality principle and the stability principle (behind the positive energy condition) beyond Lagrangian quantization.

In the construction of these new charge carriers (which result from old principles !) one has to leave the Lagrangian quantization framework and use the methods offered by algebraic QFT. One needs many new mathematical concepts and it testifies to the conceptual strength of the algebraic method, that in developing the algebraic framework of superselection rules, Doplicher, Haag and Roberts [7] whose papers were written a decade before the impressive mathematical work relating subfactors and symmetries of Vaughan Jones, anticipated (in a more limited physical context) some concepts of the latter. This profound connection was deepened in the context of low-dimensional QFT and braid group statistics [25, 8].

Looking at the rather long time of physical stagnation in the post electroweak era, one gets the impression that the Diracian method has been much more fruitful for mathematics than for physics.

Most of the more recent scientific contributions of Claude Itzykson, to whose memory I dedicate this article, are "Diracian" with geometrical ideas in the forefront.

I remember very spirited conversations on the present state and the possible future role of QFT; the last such conversation over dinner in Berlin in late autumn 1994. With sympathy and respect, it is easy to enjoy also those conversations which do not lead to an identical viewpoint. Claude listened to my account of the deep crisis in physics notably in the area of QFT. Finally he suggested to me not to worry about these inevitable developments too much, and instead enjoy the beautiful mathematical structure evolving from physical ("Diracian") ideas. Although Claude always kept a critical distance to the "physics of everything", he enjoyed certain fashionable ideas, because they gave him a chance to work on those mathematical structures, which he found so appealing without having to apologize permanently for doing mathematics instead of physics. In that respect times were more lenient than two decades before, when e.g. Swieca and I had to present an apologetic behaviour for being interested in $d = 1 + 1$ "pathologies".

2 Standard $d = 3 + 1$ versus Low-Dimensional QFT

One of the most impressive structural achievements in $d = 3 + 1$ algebraic QFT is the elaboration of the DHR superselection theory of local nets [7] which culminates in the DR-theorem [26] on the boson-fermion statistics alternative and its inexorably linked internal compact symmetry group as computable attributes of the observable algebra. One very physical way of understanding its content is in terms of a two-step process:

(i) The derivation of para-boson or -fermion statistics (mixed Young tableaux) in the sense of Green [7] from the algebraic net properties and appropriate state localization properties [7, 27]. This first step yields charge-carrying fields (elements of the "exchange algebra" or the "reduced field bundle") with R-matrix structure constants [27]. The latter define a representation of the infinite permutation group S_∞ and therefore fulfill the relation $R^2 = \mathbf{1}$ in addition to the Artin B_∞ braid group relations. The assumed state- localization properties are those suggested by Wigner's analysis of particles as irreducible positive energy representations of the Poincaré group. Not all sectors of potential interest (charged infraparticles i.e. physical electrons, infinite energy quark states) are however covered by Wigner's theory.

(ii) The mixed Young-tableaux lead to statistical weights which may be encoded into a multiplicity enlargement of the Hilbertspace and the construction of multicomponent fields (number of components = statistical weight). These tensor fields are bosonic or fermionic and transform under a compact internal symmetry group which is computable from the structure of the observable net [26].

Clearly this kind of insight cannot be obtained from a Lagrangian approach, rather it constitutes the *prerequisite* for the applicability of such an approach. Indeed, the 'para-on' fields, although having the same local observables and the same S-matrix as the tensorial fields, are more noncommutative (R-matrix instead of local bosonic or fermionic commutation relations) and less local than the tensorial fields. They do not have a (quasi-)classical limit and hence, unlike their tensorial counterparts do not fit into a Lagrangian framework [14]. Therefore the formal requirement to work with fields which can be thought of as being obtained through a quantization gives preference to the tensorial description.

In $d = 2 + 1$ QFT one still can carry out the first step (i) by using the string-localization resulting via the BF-theory [4, 24] from the mass gap hypothesis. But whereas in $d = 3 + 1$ this would still lead to exchange fields with $R^2 = 1$, this additional relation (which forces the braid-group to coalesce to the permutation group) is absent in this case. The statistical dimensions, apart from the abelian braid group situations, are now non-integer numbers, and the D.R. construction of encoding these statistical weights into multiplicities of tensorial fields i.e. the

step (ii) does not work.

It turns out that chiral conformal field theories which have much simpler analytic properties than $d = 2 + 1$ massive theories, lead to the same family of admissible superselection rules and braid group representations [9]. In both cases one observes (as a substitute for (ii)) some very fundamental looking differences from the standard $d = 3 + 1$ theories. The observable nets now allow for a natural compactification in terms of a global "universal algebra" [9]. This rather big observable algebra contains "charge measuring" operators which turn out to be dual to the "charge-creating" endomorphisms of the DHR theory. In addition the new situation is selfdual in the sense that the Verlinde matrix S, which in algebraic QFT just represents the value of the various charges in the various superselection channels, turns out to be symmetric with respect to its charge-measuring - charge-creating entries. In the case of (finite) group symmetry, this only happens for abelian groups; but for the case at hand, one does not know an internal symmetry concept which "explains" the symmetry of S and the non-integer statistical dimensions in terms of a multiplicity concepts.

One observes that the universal observable algebra $\mathcal{A}_{uni}$ contains also a kind of "charge-induction" operators which change charges, but only in the presence of other charges [14]. These operators, if inserted between basis elements of the orthogonal intertwiner basis, break the orthogonality [14] and lead to generalization of the matrix S which are associated to the representation theory of so-called mapping class groups. So it seems that the latter form an important aspect of the ill-understood new symmetry concept (a "universal" mapping class group algebra?).

Presently there is no good comprehension of why in low dimensional QFT the internal symmetry gets so inexorably linked with the space-time symmetry [14], whereas in higher dimensions one was unable to "marry" them in a nontrivial way (apart from that rather uninspiring "marriage" via extended supersymmetry). Witten's topological QFT in terms of functional integrals as well as the algebraic approach through localized endomorphism and their intertwiners (appendix of ref.) describe e.g. 3-manifolds invariants in a field theoretic setting but do not really "explain" the aforementioned phenomenon. The q-deformation theory of "quantum groups" does not cast any light on this problem either; even worse such attempts are inconsistent with the positivity of quantum theory and their only useful role seems to be to furnish admissible R-matrices at the roots of unity. The latter also appear in a much more natural and physical way (compatible with quantum theory) via the DHR or the Jones subfactor theory.

Although algebraic QFT denies the existence of a "local gauge principle" with a *direct* physical meaning, some of its attributed associated structures, as the description of charge carriers of topological charges in the sense of [7, 24] in terms of semi-infinite ("Mandelstam-like") spacelike strings or as the appearance of additional degrees of freedom at short distances [16] (the asymptotic freedom property of certain gauge theories) are naturally accounted for as an extended

realization of known physical principles. Since algebraic QFT was introduced with predominant emphasis on *structural problems* of QFT, it is particularly gratifying to note that its methods have proved to be increasingly useful for the actual *construction* of objects as anyons and plektons which hardly fit into a Lagrangian quantization framework.

Before we present some partial results in the next sections, it is worthwhile to take notice of the fact that the issue of braid group statistics [28] is essentially a quantum field theoretical problem for which the recourse to quantum mechanics (despite particle number conservation) does not yield a significant simplification.

Consider for example Wilczek's idea [29] of using the Aharanov-Bohm effect in order to obtain an interaction between pairs of "dyons". Since this interaction is extremely long range, one does not a-priori know the kind of boundary conditions to be used in the stationary scattering formalism (there is a corresponding problem in the time-dependent formulation). For physical reasons the multi-particle S-matrix *must* however *fulfill* the standard *cluster property.* If for $(n+1)$ particles one particle is converted into a "spectator" (i.e. shifted to spatial infinity), then the $(n+1)$ particle S-matrix has to coalesce into the n particle S-matrix. This consistency condition, although trivially satisfied for short-range interactions, gives rise to a "bootstrap" problem in the present long-range case. The transcription of this problem in terms of quantum mechanical path-integrals does not seem to solve this consistency problem with the cluster decomposition.

The n-particle boundary condition is only implicitly defined by this cluster requirement which couples all channels. Therefore the main advantage of the nonrelativistic limit, namely the decoupling of these channels through particle conservation is lost, and one may as well face the QFT problem of plektons right away and then construct the non-relativistic limit afterwards (since in QFT the cluster properties are safely built in).

The braid group statistics problem has a vague resemblance with relativistic integrable models of QFT (e.g. the Sine-Gordon theory) in the sense that, although there is no creation of "real" particles (the S-matrix is purely elastic), the "virtual" particle structure is quite complex.

3 More on Physical Aims

After we took notice of the fact that the problem of plektonic statistics cannot be simply solved by referring first to a "plektonic quantum mechanics" (but requires a more careful field theoretic treatment), it is worthwhile to ask about the physical aims one hopes to achieve with a theory of plektonic fields.

Most mathematically-minded field-theorist are aware of the rich applications of standard $d = 3+1$ QFT to elementary particle physics and therefore consider low-dimensional QFT as a kind of "theoretical laboratory". As a result of absence of genuine interactions on half the lightcone (Huygen's principle in $d = 1+1$

leads to chiral factorization and rules out any genuine interaction), chiral conformal QFT is a kinematical family of QFT's which realizes the complete set of admissible plektonic statistics analogous to 4-dimensional free fields.

As a generalization of analytic-structural arguments about 4-point functions [21] one finds for each admissible family of plektonic statistics (Hecke algebra plektons [8] and Bierman-Wenzl algebra plektons) two families for their field theoretic realizations. In terms of standard terminology in chiral conformal field theory they correspond to "W-models" (observable algebra without continuous group symmetry) and current-algebra models. There are also good arguments [30] that the former can be obtained from the latter by the DHR invariance principle i.e. the application of a conditional expectation through averaging over the group. The arguments that these families (apart from isolated exceptional cases) exhaust all plektonic possibilities are somewhat weaker.

It is physically very profitable to take notice of the fact that these families (beyond their role as theoretical laboratories) explain and classify the critical indices of a large class of $d = 2$ classical statistical mechanic models in terms of the statistical phases (related to braids and knots!) of the associated noncommutative chiral conformal QFT. Even though some of the Euclidean field theories, which describe the critical limit of statistical mechanics, may (modulo renormalization problem) permit an interpretation in terms of Lagrangian and path-integrals, the noncommutative chiral theory (which carries all the computational power) is not describable in such terms (the analytically continued chiral theory is as noncommutative as the original real time theory). The aforementioned analytic determination of 4-point functions based on their plektonic content does not use quantization ideas. There is the realistic hope that the critical behaviour of a large class of classical statistical mechanics models (perhaps even all) may be understood in terms of the associated noncommutative nets and their braid group statistics which is derived and classified solely on the basis of the locality principle and the stability requirement mentioned in the introduction.

Although this deep relation of the systematics of $d = 2$ critical indices with the superselection charges of chiral conformal theories has all the attributes of a miracle, this mystery can be traced back to the (still somewhat mysterious) discovery of the relation between commutative (abelian von Neumann algebras) euclidean and noncommutative (causal nets) real time field theory. The conceptual frameworks including the interpretation of localization in both theories is totally different, a fact often not appreciated by physicist who use the Euclidean formalism for other purposes (e.g. numerical calculations) than structural investigations of QFT. Whereas in $d \geq 1+1$ massive theories the analytically continued correlation functions have maximally two "physical" boundaries, namely the "Minkowski- boundary" (expectation values of noncommutative operators) and the "Euclidean boundary" (stochastic expectation values of commuting variables in the sense of classical statistical mechanics), *the restriction of the continued chiral correlation functions to* any *real one dimensional boundary define a posi-*

tive definite Wightman theory. This curious property makes chiral theories more geometric than others and permits the association to Riemann surfaces. It is (in my view) the origin of the high space-time symmetry (Möbius-invariance, diffeomorphism covariance) and the inexorable and mysterious link of space-time symmetry with internal symmetry in chiral theories (also expected for $d = 2+1$). The Riemann surfaces appear where in standard Wightman theories one had the BWH analyticity domain of vacuum correlation function. For e.g. genus one, the original real time QFT lives on one cycle and the "Euclidean" on the orthogonal cycle. The position of these cycles is not fixed and what was called "Euclidean" is now as noncommutative as the original theory.

Plektonic theories in $d = 2 + 1$ are physically more important (physics of quantum layers), but analytically more difficult than chiral conformal theories. The main reason is that their charge-carriers have a semi-infinite spacelike string localization and their superselection structure does not uniquely determine them, rather they have physical deformation parameters (coupling constants). These superselected charges may belong to $d = 2 + 1$ Wigner particles, whose nonrelativistic limits are quasi-particles of condensed matter physics (which retain their plektonic statistics in the nonrelativistic limit.).

Our present understanding of plektonic properties already supplies us with two families of numbers of great potential physical significance [9, 14]:

Algebraic QFT	$---$	**condensed matter physics**
statistical dimensions	$\longrightarrow$	amplification factors, rel. size of degree of freedom
statistical phases	$\longrightarrow$	e.m. properties of plektons, fractional Hall effect ?

A particle physicist would think of amplification factors as the analogs of Casimir's multiplicity factors ("$2I + 1$") in cross sections. But such weights for degrees of freedom also enter (and modify) the BCS relation between critical temperature T_c and the gap Δ (at zero temperature):

$$T_c = d_\rho^2 \; const. \; \Delta \tag{1}$$

Here d_ρ is the statistical weight of the plekton carrying the ρ- superselection charge. For $d_\rho = 1$ the relation is the standard BCS relation. The quadratic dependence on d_ρ can be made plausible for a hypothetical SU(2) fermionic BCS model (linear dependence on the Casisimir eigenvalue 4). Anyons (=abelian plektons) have $d_\rho = 1$ and change only the statistical phases, but do not amplify the weights.

Note that non-abelian symmetry groups in condensed matter physics can only appear in an artificial way i.e. by making ad hoc approximations on the $U(1)$-invariant many-particle problem. On the other hand, since plektons are as selfconjugate as the superselection structure of abelian group symmetry, a phase transition from a $U(1)$ liquid fermi phase to a plektonic phase is perfectly conceivable. It cannot be stressed enough, that the issue of relativistic localization is only important in the classification of physically admissible plektonic statistics.

As for Fermions and Bosons the results remain valid in the nonrelativistic domain. Since, as already mentioned, the relation between the superselection data and the fields in $d = 2+1$ is not unique, we need a principle which selects one realization among all dynamically possible ones (i.e. a substitute for a Lagrangian). Such a principle based on the idea of "freeness" will be abstracted from the long-range cluster properties of the S-matrix in what follows.

In $d = 3+1$ the S-matrix fulfills the well-known cluster properties which in the extreme cluster limit (all pairs become infinitely separated in the sense of their wave-packet localization) takes the form

$$S \overset{cluster}{\underset{limit}{\longrightarrow}} 1 \tag{2}$$

It is believed that the Borchers classes of the various free field exhaust the possibilities of local interpolating fields with trivial scattering, however a solid proof only exists for zero mass field [31]. If true, this would select the various standard $SU(N)$ or $SO(N)$-selfconjugate free fields of $d = 3+1$ QFT as the "freest" Bosons or Fermions.

For $d < 3+1$ this limit generically lead to $S_{lim} \neq 1$. In particular a more detailed analysis shows [14] that for $d = 1+1$ one obtains (as the leading long-distance contribution) an elastic energy (rapidity) dependent S-matrix which fulfills the Yang-Baxter equations as a consistency condition. The only surprise is that such a limiting S-matrix is again the S-matrix of a localizable QFT, the starting point of the so-called "bootstrap program" for the the construction of relativistic integrable $d = 1+1$ massive models.

In $d = 2{+}1$, S_{lim} seems to be piecewise constant in momentum space [14] with matrices which jump from one R-matrix (representation of a word in the braid-group) to another as soon as the particle velocities go through a coalescent degenerate configuration. The cross sections of such piecewise constant S-matrices are expected to vanish and there is some ambiguity in what part of the asymptotics should be accounted for as a change of inner products for in- and out-states, and what part should belong to the proper S-matrix. In any case, the localized fields which may correspond to such a situation have not yet been computed. In section 6 we use an extension of the representation method of Wigner (which is however limited to anyons) in order to shed some light on the localization- and statistic-properties of "free" plektonic particles and fields.

4 Remarks on Perturbation Theory and Interactions

Standard perturbation theory aims at the perturbation expansion for the vacuum expectation values of (time ordered) products of point-like quantum fields. The most convenient presentation is via Feynman rules, which can be formally derived

either by operator- or by functional methods. The rules for renormalizing those formal expressions have remained complicated (compared to the simplicity of the Feynman rules themselves) and an elegant incorporation of renormalization into the operator or functional method does not exist for good reasons (see below). The separation of the local algebraic aspects from the global state properties can in principle be done at the end (assuming that the perturbation series converges in some sense) with the help of the (suitably adapted) GNS reconstruction. But in practice this remains a very difficult task and this last reconstructive step is usually left out; sometimes in the erroneous belief that functional integrals based on classical actions will (at least on a formal level) always define *quantum* field theories.

The algebraic approach to perturbative interactions, even on a formal level, organizes things in a slightly different way. The first step is the construction of a net in terms of free fields in Fock-space. The choice of the Fock-space is determined by what kind of free fields one wants to couple. For implementing interaction one uses Bogoliubov's operator [32] $S(g.h)$ in Fock-space which has formal unitary representation terms of time ordered products:

$$S(g,\underline{h}) = Te^{-i\int(W_I(\underline{\varphi}_0(x))g(x)+\underline{\varphi}_0(x)\underline{h}(x))d^4x}. \tag{3}$$

Here $\underline{\varphi}_o(x)$ stands for the collection of free fields and W_I is a Wick-ordered L-invariant (polynomial) coupling. S was used by Bogoliubov to introduce causal fields in Fock-space:

$$\underline{\varphi}_g(x) = S^{-1}(g,h)\frac{\delta}{\delta\underline{h}(x)}S(g,h)\Big|_{h=0} \tag{4}$$

Here g is a Schwartz test function which is chosen constant on a large double cone $\mathcal{C}$.

It is then a simple consequence of Bogoliubov's causality property [32] that $\varphi_g(x)$ is independent of the values of g outside the past light cone $V_-(\mathcal{C})$ subtended by $\mathcal{C}$. According to a recent observation by Fredenhagen (private communication, to be published) a change of g inside $V_-(\mathcal{C})$, which leaves the constant value in $\mathcal{C}$ unchanged, can be implemented by a unitary transformation U in Fock-space:

$$U\underline{\varphi}_g(x)U^{-1} = \varphi_{g_1}(x) \quad x\in\mathcal{C} \qquad , g_1\Big|_{\mathcal{C}} = g. \tag{5}$$

Since the local net *within* $\mathcal{C}$ does not change under a common unitary transformation (of all of its members), the fields $\varphi_g(x)$ are formal candidates for "field coordinates" of a net in $\mathcal{C}$. $\mathcal{C}$ can be made arbitrarily large (the possible divergence of the U's in (5) pose no problem for the net), and hence one obtains a net of algebraic QFT, this time not in the vacuum representation but rather in an auxiliary Hilbert space. States on this net, in particular the vacuum state, lead

to physical representations of the global C^*-algebra which are inequivalent to the auxiliary Fock-space representation.

Several comments on this procedure are helpful. In order to get away from the formal perturbative time ordered expressions, Bogoliubov axiomatized the properties of the test-function dependent unitary operators $S(g, \underline{h})$ in Fock-space [32]. It seems that for the distinction between different interaction polynomials one needs some smoothness of these S in terms of their dependence of $\underline{f}$ and $\underline{h}$. Even presently it is not known if there exist such operators in Fock-space which correspond to a given W_I, i.e. whether the Bogoliubov axiomatics has such solutions.

The second step, namely the construction of the vacuum (and other) representation is related to the so-called "adiabatic limit". This two-step separation avoids the pitfall related to Haag's theorem [7] ,which can also be circumvented by the conceptually less attractive introduction of an infrared cutoff and the construction of the thermodynamic limit (the adiabatic limit). The chances for existence of the net together with a vacuum state on it may even improve if one restricts the Bogoliubov method to a smaller algebra e.g. the algebra generated by neutral composite fields. An example is the coupling of a massive vector meson to a conserved spinor current for which the neutral subalgebra stays renormalizable.

It is interesting to note, that even on a formal level it is not clear, that gauge theories fit into this two step perturbative construction. The difficult point here is that the gauge condition which are necessary for the definition of operator algebras (e.g. the Gupta-Bleuler condition) are mostly global, even in abelian gauge theories. The net formulation however is only consistent with local conditions.

One may hope that one should be able to bypass the existence problem of the Bogoliubov axiomatics by constructing "interacting wedge algebras ". This hope is based on the recent observation that the free fields algebras belonging to the ("Rindler" or "Bisognano-Wichmann" according to upbringing and taste [7]) wedge region can be directly constructed without reference to local fields [39]. This raises the expectation that there should also be a truly algebraic way to introduce an interaction for this preferred localization region. Support for this hope comes from the distinguished representation of the S-matrix in terms of modular conjugations:

$$S = J_W \cdot J_{0,W} \tag{6}$$

Here J_W is the modular conjugation of the interacting wedge algebra, whereas $J_{0,W}$ has the same relation to the "incoming" interaction-free wedge algebra. This formula is only a transcription of the representation of S in terms of the corresponding $TCP = \theta$ operators:

$$S = \theta \cdot \theta_0 \tag{7}$$

Here we used the Bisognano-Wichmann relation between θ and J_W which are

only different by a π-rotation around the x-axis:

$$\theta = U(R(e_x,\pi))J_W \quad , \quad W = tx\text{-wedge} \tag{8}$$

as well as the action of θ on $\underline{\varphi}$ and $\underline{\varphi}_{in}$

$$\theta\underline{\varphi}(x)\theta = \cdot\underline{\varphi}^*(-x) \tag{9}$$

hence $\theta\underline{\varphi}_{in}(x)\theta^{-1} = \underline{\varphi}^*_{out}(-x)$

$$\theta\theta_0\underline{\varphi}^*_{in}(-x)\theta_0\theta = \underline{\varphi}^*_{out}(-x) \tag{10}$$

and hence (6) up to a phase factor which may be absorbed into the θ's. Therefore a natural distinguished equivalence transformation between the interacting and the free (incoming) wedge algebra would be given by a "square root" of S (which is expected to be a kind of field theoretic Møller operator). Unfortunately we do not have the vacuum representation, but rather a representation of the wedge algebra in some auxiliary Fock-spaces which is related to the vacuum representation by another unknown unitary equivalence transformation. A possible construction of interacting nets in the Hilbertspace of incoming states based on this distinguished role of the S-matrix as a modular invariant has been sketched in [40].

It is very gratifying to notice that the von Neumann algebras of this "perturbative" net (i.e. induced by perturbative couplings between free fields) belongs to a folium of states (the reference state and all the mixed states obtained by density matrices constitute a folium) on the local algebras (double cones and wedges) which allows an elegant characterization in terms of a "microlocal spectrum condition" [13]. Whereas the usual spectrum condition (vacuum and positive energy condition) is a global requirement, the microlocal spectrum condition is a condition which can be formulated directly on the local algebras. It is on the one hand weaker (it characterizes a whole folium of states instead of a unique vacuum ground state and it does not distinguish superselection sectors), but on the other hand it promises to lead to an intrinsic local characterization of interactions [13].

As mentioned in the introduction, this microlocal spectrum condition was recently discovered in the quantum field theory in curved space-time for which the ground state (or vacuum) requirement becomes meaningless (apart from some special cases which admit a time-like Killing vector). As far as the curved space-time theories are concerned, the microlocal property not only led to an understanding of the "Hadamard recipe" (and its equivalence to the so-called adiabatic reference state construction) [13], but it also permits an incorporation of interacting matter (nonlinear equation of motions) and a natural extension of the Wick-products and the definition of an energy momentum tensor operator. The latter is needed in a quasiclassical interpretation of the classical Einstein equations.

Hence the algebraic method suggests new ways of looking at old problems. In this way one also hopes to get a better understanding of which difficulties in

perturbation theory already occur on a local level (i.e. in the construction of nets) and which of those are more related to global obstructions (i.e. related to states). The free fields used for the construction of the interaction and the local net become more dissociated from the physical particle content of the model, so that incorrect pictures about particles being closely related to fields are avoided.

The quantization approach via functional integrals is entirely global since it is physically meaningless to functionally integrate over the field space belonging to a local piece of Minkowski-space. In addition such a formulation is only meaningful in the framework of a subclass of Euclidean theories and one has to keep in mind that the Euclidean localization (in the sense of statistical mechanics) is extremely nonlocal with respect to the real time localization.

The end of this section is also the natural place to make some more detailed comments about the most serious problem in contemporary QFT and elementary particle physics: the profound crisis of the last twenty five years. Even for somebody not familiar with the content of the various fashions the crisis is already evident from the observation that nothing of any physical relevance happened after the discovery of the electro-weak interactions (except its detailed experimental verification). This stagnation is very remarkable in view of the fact that the first three quarter centuries were so rich in this respect. A glance at the selected papers edited by J. Schwinger ["Selected Papers on Quantum Electrodynamics", edited by J. Schwinger, Dover Publications 1958] or even the later papers edited by Imai and Takahasi [QFT I and II in "Series of Selected Papers in Physics] removes any doubt. Even papers on the most central issues of contemporary physics as the recently reported (even in the international press) breakthrough in QCD confinement based on the fascinating ideas concerning electric-magnetic duality seems to be without much computational consequences and creates the feeling of looking at a faint shadow of a theory. Since effective Lagrangians are just names for certain momentum space Taylor coefficients of correlation functions, the relation to local quantum physics of these global observations remains unclear[6]. This state of affairs which is so starkly different from the aforementioned situation, cries out for an explanation which certainly has nothing to do with the intellectual sophistication of the involved physicists.

It would be to easy to blame this crisis on the marketing of string theory, although statements of some of the proponents at conferences [7] may tempt one to do so. But things are not that easy. String theory is *not* the cause for the crisis but rather one of its most remarkable manifestations. In my view the roots of the present crisis can be treated back to the aftermath of the early renormalization theory, i.e. to one of the most important cross roads of modern physics. The infinities which appeared in QED are not "intrinsic", they are rather the

[6]In one of those papers [15] the author seems to suggest that in order to secure these duality observations, one may need a different conceptual framework.

[7]"The string theory allows to understand everything, but compute nothing" (David Gross's answer to the question of an experimentalist about the computational aspect of TOE)

result of the repair job one has to perform on a slightly incorrect "quantization" (Lagrangian) starting point. The idea of Kramers concerning the distinction of physical from "naked" parameters (the self-energy problem) which came from the classical Poincaré - Lorentz electron theory was extremely important in order to disentangle the physical (renormalized) answers [44]. However these ideas becomes somewhat of a burden (like many good ideas) if misread as a kind of parallelism between classical and quantum field theoretical ideas about the electron. In QFT particles cannot be introduced in addition to fields, but they are consequences of local relativistic fields. Wigner's characterization of particles as irreducible representations of the Poincaré group is already an inexorable part of the unitary representations coming with quantum fields or algebraic nets. Whereas in the classical case one had to add the particle structure via those partial models of Poincaré and Lorentz, in local quantum theory there is no such possibility because it is already there. Therefore there should be an alternative formulation which allows to avoid those infinities. Indeed the "split point method" first used by Schwinger and Johnson can be used to accomplish this. As one can define free field Wick products instead of using (global) frequency ordering also by local limits through formulas involving point split products, one may similarly define composite normal product fields which enter the field equations as the interacting terms. An illegitimate interchange of the limit with the additive and multiplicative components of such a normal product expression will bring back the infinities. Since the interaction graphical rules are not as elegant (there are several types of two-point functions and not just one type as the time ordered propagators in Feynman's scheme) such renormalization schemes never appeared on a sufficient general level in the literature; only in simple two-dimensional models as the Thirring or Schwinger model they have been used. In general it is much faster to compute with Feynman rules and repair the illegitimate start by regularization and renormalization.

The wrong turn came about by interpreting the subtle and deep relation between classical statistical mechanics and noncommutative QFT too naively. Whereas statistical mechanics has natural physical cutoffs, this is not the case in QFT. In order to appreciate this remark without prejudices, it is helpful to take a look at the fascinating history of attempts at relativistic *nonlocal* theories and at the notion of elementary length. In the 50^s it was thought that by putting form factors into the interaction Lagrangian one could avoid the above infinities and encounter a less singular behaviour in correlation function. But it was soon realized (Kristensen) that through their iteration such form factors have a disastrous effect on macro-causality; their presence wrecks the physical interpretability of the theory. Later Lee and Wick made the proposal to change Feynman rules by pairs of complex conjugates poles (which would also "soften" the light cone singularities) again with the same unacceptable result of a-causal precursors as was shown later. If a cutoff in relativistic QFT would be more than a formal intermediate device without any intrinsic physical significance, this would have

been a truly sensational event. The increasing number of nontrivial massive local QFT's (Ising QFT, Sine-Gordon QFT,...) which have been constructed via the nonperturbative formfactor program (starting from factorizing elastic S-Matrices) have significantly reduced the space for folkloric ideas on renormalization. More specifically the following ideas turned out to be prejudices [8]

(i) quantum gravity has to be invoked in order to make QFT well-defined.

(ii) Light cone singularities which go significantly beyond the canonical behaviour threatens the existence of quantum fields.

Anybody who believes that a cutoff has a physical significance in a real time QFT should try to modify one of those low-dimensional models by the implantation of a relativistic cutoff *without wrecking* the physical *interpretability* of the model. All structural investigations of QFT tell us that without Einstein causality the basis for the physical interpretation of the theory is lost e.g. there would be no basis for the validity of cluster properties, scattering theory etc.. Time and again we had to learn that notions like "a little bit nonlocal" (as well as a "little bit nonunitary") are as nonsensical as "a little bit pregnant", at least within the accepted framework of QFT. Only if one goes completely outside standard QFT into the direction of *noncommutative space-time* one finds a chance for replacing causality by some new structure [52] whose consistency is presently under investigation.

It is interesting to note that for certain families of lattice theories, cluster properties and hence the existence of multiparticle ("multi-magnon") scattering states can be shown; but those are of course not relativistic and moreover the proofs are *much* more involved than the corresponding derivations in local QFT. Needless to say that my critical remarks naturally include one of my own papers: I nowadays consider my contribution on the 4-ε expansion to the enhancement of physical knowledge as being smaller than any preassigned ε [45].

I will not follow here the path which leads from the "original sin" of neglect of the "intrinsicness" of concepts in QFT to the unscrupulous treatment of established physical principles in string theory which in my view the path into the present crisis.

Rather I would like to mention some modest successes outside this way of thinking. Closely related to the ambitious program of finding non-Gaussian fixed points of the renormalization group is the more modest program of discovering the physical concepts which determine the spectrum of admissible critical indices in $d = 2$ (i.e. the generalization and extension of Onsagers work on the Ising model). Starting from Kadanoff's "Coulomb Gas Representations" of critical indices for specific family of models and going through the discoveries of Belavin, Polyakov and Zamolodchikov of the minimal models (associated to the Virasoro algebra), this program has now reached a certain conceptional depth through

[8] The idea that theories which are "finite" in the sense of Feynman perturbation theory may also be physically preferred (supersymmetry, string theory) results from this prejudice.

the recognition of critical indices as being the numerical values of superselection charges related to the "statistical phases" of algebraic QFT. The latter relates the spectrum to braids and knots. It seems that the issue of braid group statistics is susceptible to a complete classification, at least in "rational" chiral theories (the analogy with the classification of finite depth Jones subfactors is more than superficial).

The modern point of view which analyses chiral conformal QFT directly from the local field theoretical principles (without the intervention of global (Fourier) algebras as the Virasoro- or Kac-Moody-algebra) remained remarkably close to the QFT framework in which the present author and collaborators came across conformal QFT in 1974, an observation which I already elaborated in ref.[14]. My construction in 1974 [47] of the chiral energy momentum tensor as a "Lie-field" was inspired by older work on Lie fields starting 1961 with Greenberg and culminating in the 1967 work of Lowenstein [48]. These Lie fields in present terminology are (not necessarily conformal) W-fields, i.e. conformal causal fields for which (modulo a c-number Schwinger term) the commutators close. The case of a one component scalar field was most extensively studied [48], but the examples, although structurally close to current algebras (bilinear in free fields), were physically not as interesting as the chiral energy momentum tensor [47], not to mention the more recent development on W-algebra. With the additional knowledge of the differential identities [47] between fields and current (corresponding to the modern Knisknik-Zamolodchikov identities) and the old conformal decomposition theory[9] the question arises, why was the richness of chiral conformal field theories (minimal models etc.) not seen already at the beginning of the 70's? Partly the reason was in my opinion that the confidence with which physicist investigate low dimensional QFT nowadays was lacking at that time. Contributions in low-dimensional field theory were often "buried" in conference reports since they did not enjoy much appreciation. Another perhaps even more important reason is that a systematic mathematical study of representation of infinite dimensional algebras was not yet available. Although mathematicians use more global methods (Fourier coefficients of T's and j's), the (global) Verma module approach to unitary representations was important also for "local quantum physicist" since it generated the missing mathematical confidence. The rupture between the "old" physically oriented and the "new" geometrically inspired QFT occurred during the second half of the 70's. THis in my view unique phenomenon in the history of physics is one of contributing factors to the present crisis [14].

Recently a clever strategy of "perturbing" chiral conformal QFT has been found by Zamolodchikov. Here "perturbation" is not taken literally (otherwise one would have to handle terrible infrared divergencies), but rather serves as the starting point for consistency arguments which select the "integrable" massive representatives. These representatives are probably identical to the theories

[9] For a comparitative study of the old versus the new approach see ref.[14].

which correspond to the large distance limit of the S-matrix in the sense of section 3. There remains the interesting question of the number of massive superselection classes (according to section 3 equal to the number of factorizable models) which correspond to one conformal limit. For example it is known that the conformal Ising model allows for two different integrable perturbations and there are indications that these two possibilities are already preempted through two different interpretations of the Ising partition functions [10]. These are open problems where one expects algebraic field theory to contribute.

It is interesting to note that Zamolodchikov's [46] construction picture runs opposite to the renormalization group extrinsic approach. Our starts in a completely intrinsic matter from a classification of rather simple theories (as sketched above) and moves away towards more complicated theories by applying structural self-consisting arguments on relevant "perturbing" fields which are chosen from the list of composite fields of the chiral theory. It seems that for practical purposes the way from simple models to more complicated ones is more constructive than the renormalization group approach. The latter is too structureless for (I have not seen any construction of a non-Gaussian fixed point except the ill-fated $4-\varepsilon$ expansion). But on the other hand the algebraic QFT point of view (and Zamolodchikov's approach, which is very close) is limited to $d = 1+1$, even though it uses concepts which are quite general. There is no reason to believe that the superselection structure will determine critical indices in $d > 1+1$.

Some of the remarks made in this section should be viewed as a qualification of my critical comments made at a round table discussion "Physics and Mathematics" [XIth International Congress of Mathematical Physics, International Press, Daniel Iagolnitzer Editor].

5 Duality- and Modular- Properties of Chiral Field Algebras

It has been known for some time in algebraic QFT, that certain nontrivial inclusions of observable algebras in the vacuum representation contain valuable informations about superselected charges. In the following we will describe three different type of inclusions.

(i) *The "corona-inclusion" [III. 4.2 of ref. 7] in $d = 3+1$ massive QFT with a mass gap*

$$\pi_0(\mathcal{A}(K_1) \vee \mathcal{A}(K_\infty)) \subset \pi_0(\mathcal{A}(\mathcal{C}))'. \tag{11}$$

Here K_1 is a centrally located-double cone which is surrounded by a double-cone-ring $\mathcal{C}$, and K_∞ is the ring-like wedge region consisting of all points which are space-like relative to $\mathcal{C}$ and K_1. With other words $K_\infty \cup K_1$ is causally

[10] I am indebted to A.A. Belavin and A. Fring for discussions on this point.

disjoint from the ring $\mathcal{C}$. The operator notation is standard: π_0 is the vacuum representation and the upper dash denotes the commutant of an operator algebra.

In case of the existence of only a finite number of DHR superselection sectors, the DR theorem tells us that this inclusion is equivalent to that defined by the fixed point algebra under a "double" of a finite group G ($\mathcal{F}$ is the DR field algebra):

$$\begin{aligned} \pi_0(\mathcal{A}(K_1)) \vee \pi_0(\mathcal{A}(K_\infty)) &= (\mathcal{F}(K_1) \vee \mathcal{F}(K_\infty))^{G\times G} \mid \mathcal{H}_0 \\ \pi_0(\mathcal{A}(C))' &= (\mathcal{F}(K_1) \vee \mathcal{F}(K_\infty))^{Diag(G\times G)} \mid \mathcal{H}_0 \end{aligned} \tag{12a}$$

$$(\mathcal{F}(K_1) \vee \mathcal{F}(K_\infty))^{G\times G} \mid \mathcal{H}_0 \subset (\mathcal{F}(K_1) \vee \mathcal{F}(K_\infty))^{Diag(G\times G)} \mid \mathcal{H}_0 \tag{12b}$$

In other words the DR group symmetry of $d = 3 + 1$ charge-carrying fields is already visible on the level of neutral observables through (11). The equality holds if instead of observables we use field algebras, in particular for free fields.

(ii) *The "quarter circle inclusion" of* $d = 1$ *chiral conformal QFT.*

$$\pi_0(\mathcal{A}(I_1) \vee \pi_0(\mathcal{A}(I_3)) \subset \pi_0(\mathcal{A}(I_2) \vee \mathcal{A}(I_4))' \tag{13}$$

Here $I_i, i = 1...4$ are four intervals obtained by equipartitioning the circle (i.e. the localization space of chiral conformal theories). This inclusion is selfdual[11] in the sense that it is equivalent to the inclusion obtained by exchanging I_1 and I_3 with I_2 and I_4. Therefore it is not surprising that, apart from finite abelian groups (we again restrict our interest to the case of a finite number of superselection rules), the "quantum symmetry" of superselection sectors of chiral theories is not covered by groups. This is the case which we will treat in this paper.

The theories with a mass gap $d = 2 + 1$ fulfill a similar formula [9] with the intervals being replaced by spacelike cones.

(iii) *The "e.m.-corona. inclusion" for* $d = 3 + 1$ *Maxwellian-like free fields.*
The formula is the same as (11) i.e.

$$\pi_0(\mathcal{A}(\mathcal{C})) \subset \pi_0(\mathcal{A}(\mathcal{C}'))' \tag{14}$$

but now $\mathcal{A}(\mathcal{C})$ is the algebra generated by zero mass neutral spin 1 fields (electromagnetic fields). In particular the chargeless free electromagnetic fields leads to a proper inclusion [7] without nontrivial superselection sectors which one could blame for this duality obstruction. The equality sign can be formally recovered by using indefinite metric gauge potentials, but the use of unphysical gauge fields in

[11] It is the chiral analogue of the corona inclusion.

indefinite metric spaces is against the spirit of algebraic QFT. The latter prefers to deal directly with the cohomological problem hidden behind the proper inclusion instead of artificially removing it at the expense of introducing vector potentials.[12] For more details on this inclusion see ref. [40]. It is believed [7] that the existence of nontrivial electric and magnetic charges (i.e. Maxwell-like interactions) will remove this obstruction against Haag duality. It is interesting to note that the order-disorder duality structure in low dimensional QFT does not permit the simultaneous absence of order and disorder charges. Due to the unsolved infrared problems of algebraic QFT, this subject has not yet been thoroughly investigated.

I believe that a local algebraic approach to electro- magnetic duality (i.e. between $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ as in the Seiberg-Witten models) and in particular the construction of the e. and m. charge- carrying operators which are responsible for this duality, should start from these observations. Superselection rules arising from F and $\tilde{F}$ are expected to be very peculiar and related to the problematization of the notion of "magnetic fields" and quark confinement. We will not pursue this matter here.

We now return to the quarter circle situation (ii). In that case one proves the following theorem.

Theorem 1 [33] *The vacuum representation of the abelian current algebra (multicomponent Weyl algebra) violates Haag duality for the quarter circle inclusion and fulfills instead the following "lattice duality":*

$$A_L(I_1 \cup I_3)' = \mathcal{A}_{L^*}(I_2 \cup I_4) \tag{15}$$

The von Neumann algebras on both sides are generated from Weyl algebras over real test functions of subspaces of $C^\infty(S^1)\otimes V$. Let $\mathcal{S}_L(I_1 \cup I_3)$ denote the subspace consisting of real test functions which are constant in I_2 and I_4 and fulfill the condition $f(z_2) - f(z_4) \in 2\pi L$, $z_{2,4} \in I_{2,4}$ where $L=$ even lattice in V. Then:

$$\mathcal{A}_L(I_1 \cup I_3) = Alg\{W(f), f \in \mathcal{S}_L(I_1 \cup I_3)\}$$

and similarly for $\mathcal{A}_{L^*}(I_2 \cup I_4)$ with L^* being the dual lattice to L.

Note that only in the case of a selfdual lattice one obtains the standard form of Haag duality.

The content of the theorem and its proof becomes clearer if one maximally extends the multicomponent Weyl algebra. Here "maximally" means that no further (bosonic) local extension is possible. It is well-known that such maximal extensions correspond to even lattices in the sense that the functions $\underline{f}$ in the Weyl generators $W(\underline{f})$ fulfill quasi-periodic lattice boundary conditions on S^1

[12] Although vector potentials can be avoided in all structural arguments of algebraic QFT, they do enter in the Lagrangian quantization approach and in the intermediate step of the ensuing standard perturbation theory of physical observables.

instead of being periodic. This more general affine set of functions still maintains the causal net structure, but the extended net $\mathcal{A}_L^{ext}$ has a finite number, namely $|L^*/L|$ superselection sectors, instead of (continuously) infinitely many. Collecting the results in terms of theorems we have the following:

Theorem 2 [33] *The maximally extended observable net $\mathcal{A}_L^{ext}$ fulfills the following extended duality relation*

$$\mathcal{A}_L^{ext}(I_1 \cup I_3)' = \pi_L(\mathcal{F}_{L^*}^{(0)}(I_2 \cup I_4)). \tag{16}$$

Here the left hand side is the commutant of the L-extended observable algebra in its natural π_L-vacuum representation. The right hand side is the π_L-vacuum representation of the charge-anticharge split (total charge neutral, indicated by the (0)-superscript) subalgebra of the L^*-field algebra.

The proof of the second theorem is based on the first one. Both proofs are easy and natural if one adapts the framework of Buchholz, Mack and Todorov [34] and will not be given here. This extended duality is the prerequisite for a geometric form of the Tomita-Takesaki modular theory for the quarter circle situation. Here we will only present an application of that theory to the case of *no* nontrivial superselection rules (i.e. for selfdual lattices or for the CAR fermion algebra). Let us take the special case of chiral CAR algebras.Then we have:

Theorem 3 *(based on Araki's analysis of CAR algebra [35]):*
Let $U(t) = e^{iht}$ be a one parametric group of diffeomorphism acting as Bogoliubov automorphisms on the wave function space underlying the CAR (S^1) C^*-algebra. Then there exists an associated β-KMS state. If the diffeomorphism has a finite number of fixed points, then this KMS state has a natural factorial III_1 restriction to a subalgebra.

In this way one obtains a geometric modular theory for this subalgebra and a suitable state, i.e. there exists a state which together with the subalgebra has the given diffeomorphism group as the Tomita-Takesaki automorphism group.

Instead of showing how a proof can be abstracted from Araki's work, we mention two physically interesting special cases of diffeomorphisms.

$\alpha)$ $z \to Dil(t)z$ as the diffeomorphism.
In this case the situation of the theorem coalesces for $\beta = 1$ with the modular theory of the semi-circle CAR $(\cap)$ algebra and the vacuum state vector. For $\beta \neq 1$ the restricted KMS state belongs to another folium of the simple C^*-algebra CAR $(\cap)$ different from the vacuum folium.

$\beta)$ $z \to T_2^{-1}\ Dil(t)T_2$, with $T_2 : z \to z^2$ covering of S^1
This diffeomorphism has four fixed points and belongs to the quarter-circle situation. Instead of the semi-circle CAR algebra one now considers the one belonging to I_1 and the state vector is not the vacuum but it is rather defined implicitly by the GNS construction within Araki's CAR theory If the vacuum folium contains

a $\beta = 1$ KMS state for this diffeomorphism group acting on CAR (I_1), then the state must be the so constructed one. So in order to maintain a geometric modular theory for general diffeomorphisms, one should allow reference states which are different from the vacuum (and outside the vacuum sector).

To recover Haag duality in its original form [7], we must construct the field-algebra $\mathcal{F}_{L^*}(S^1)$ for the extended current algebra model. This algebra contains all localizable charge carries and it fulfills the following twisted Haag duality.

Theorem 4 [36] $\quad \mathcal{F}_{L^*}(I) = \mathcal{F}_{L^*}(I')^{tw}$
Here the superscript on the right hand side denotes the "twisted" commutant. In our case of $Z_{|L^*/L|}$-abelian models, the twist is a generalization of the fermionic twist namely a suitable "square root" of the rotation $e^{-2\pi i L_0}$[35].

Up to now we have been dealing with abelian ("anyonic") representations of braid group statistics (level 1 multicomponent current algebras). How does one construct plektons? Since a "quantum symmetry" generalizing the compact group symmetry does not (yet) exist, a conservative substitute for the previously discussed abelianization of "para-ones" which leads to bosons and fermions (i.e. the DR theorem) would be to try to "anyonize" plektons. In the following we will sketch such an attempt.

Let us start again from the multicomponent current algebra. It is well-known that the current algebra in the Weyl form defines an orthomodular functor [37] from real Hilbert subspaces $\mathcal{H}_R$ of the multicomponent complex wave function space $\mathcal{H} = L^2(S^1) \times V$ into von Neumann subalgebras $\mathcal{N}$ of $B(\mathcal{H}_{Fock})$.

$$\mathcal{H}_R \longrightarrow \mathcal{N}(\mathcal{H}_R) \tag{18a}$$

$$\mathcal{H}'_R \longrightarrow \mathcal{N}(\mathcal{H}'_R) = \mathcal{N}(\mathcal{H}_R)' \tag{18b}$$

Here $\mathcal{H}'_R$ denotes the *symplectic complement* of $\mathcal{H}_R$ with the symplectic form $\sigma(\underline{f}, \underline{g})$ being given by the imaginary part of the scalar product.

$$\sigma(\underline{f}, \underline{g}) := Im(\underline{f}, \underline{g}) = \frac{1}{2\pi} \int_0^{2\pi} < \underline{f}(\theta), \underline{g}'(0) > d\theta \tag{19}$$

$\mathcal{H}_R$ is called standard if $\mathcal{H}_R \cap i\mathcal{H}_R = 0$ and $\mathcal{H}_R + i\mathcal{H}_R$ is dense in $\mathcal{H}$. In that case $\mathcal{N}(\mathcal{H}_R)$ turns out to be a *factor in standard position* with the vacuum state vector in Fockspace being a cyclic and separating reference vector.

If the real subspace consists of real multicomponent functions localized in an interval $I \in S^1$, the T.T.- modular theory is geometric and the modular automorphism is the dilation which leaves the endpoints fixed, whereas the modular conjugation is an antilinear "reflection" of the algebra $\mathcal{N}(\mathcal{H}_R(I))$ onto $\mathcal{N}(\mathcal{H}_R(I'))$. Both modular operations are the functorial image of operators in $\mathcal{H}$. Note that *localized* real subspaces are automatically standard. This is the wave function pre-version of the Reeh-Schlieder theorem for the localized Weyl-subalgebras acting on the vacuum. It also may be viewed as a property of coherent states in Fock space affiliated with localized real wave functions.

The previous maximal local extension by an even lattice amounts to an extension of this functor to real "affine" spaces":

$$\mathcal{H}_R^{aff} \subset \mathcal{H} \times L \tag{20}$$

Here L is an even lattice which appears in the quasiperiodic boundary conditions of the real multicomponent wave function. The extended symplectic bilinear form is

$$\sigma(\underline{f},\underline{g}) = \frac{1}{4\pi} \int_0^{2\pi}(\langle \underline{f}(\theta), \underline{g}'(\theta)\rangle - \langle f'(0), g(\theta)\rangle)d\theta + \frac{1}{4\pi}(\langle f(0)\cdot g(2\pi)\rangle - \langle f(2\pi), g(0)\rangle)$$
$$f(2\pi) - f(0) = 2\pi\ell_f, \quad g(2\pi) - g(0) = 2\pi\ell_g \tag{21}$$

where $\ell \in L$ is a lattice vector which is determined by the quasiperiodic boundary conditions.

So the original Hilbertspace $\mathcal{H}$ becomes augmented by a lattice and the extended functor defines a map from real affine subspaces to von Neumann algebras which are subalgebras of the charged field algebra. The latter is a kind of crossed product between the previous neutral observable algebra and an abelian group algebra representing L. The localized subalgebras $\mathcal{A}_L(I)$ correspond to rather complicated real affine wave function subspaces [34] which are not linear subspaces.

In this sense the aforementioned maximally extended multicomponent current algebras are still interpretable as *extended* Weyl algebras.

We again emphasize that $\mathcal{H}^{aff}$ is *not* simply the tensor product of the CCR with the rotational (lattice) C^*-algebra, but rather the lattice acts on the localization function via boundary condition.

Within this functorial calculus one can form diagonal generators in the k-fold tensor product:

$$W^{(k)} = W^{ext}(\underline{f}) \otimes ... \otimes W^{ext}(\underline{f}) \tag{22}$$

They generate a subalgebra $\mathcal{A}_L^{ext,(k)}$ of the k-fold tensor product

It is well-known that the level one $SU(N)$ loop group representation is generated by the extended Weyl algebra for a particular lattice L (the root lattice of $SU(N)$). Let us therefore reinterpret the above tensor-product formula for those special group lattices as:

$$\hat{W}^{(k)} = W^{(1)}(\underline{g}) \otimes ...W^{(1)}(\underline{g}) \quad \text{with } \underline{g} \in \text{ loop-group.} \tag{23}$$

with $W^{(1)}(\underline{g})$ = loop group representation of level 1. Then $W^{(k)} \subset \hat{W}^{(k)}$.

By *reduction* we now could construct higher level loop vacuum representation which are known to lead to plektonic (nonabelian) nets. But such an ad hoc procedure invoking the loop group and its associated net is not quite in the spirit of algebraic QFT. I believe that at least for nets with a finite number of superselection sectors ("rational" chiral theories) there exists a natural plektonic extension

of the tensor product algebra generated by the $W^{(k)}$'s in formula (22). My conjecture is based on the observation that the loop group nets and the W-nets are the only irreducible plektonic families which implement the classified statistics families, a remark which generalizes the structural discussion of plektonic 4-point functions in [37].

We apologize to the reader for these complicated technicalities. They were only meant to support the hope that chiral conformal QFT can be classified by purely intrinsic QFT principles (valid in *any* dimension) without reference to particular algebras as Kac-Moody or W-algebras. Such messages are important if one believes that the main physical role of chiral theories is that of a theoretical laboratory for general QFT. Algebraic QFT is designed to cope with nets which are far away from the standard CCR and CAR situations (and their perturbative interactions). The latter can be considered as function spaces with a nonabelian C^*-structure i.e. as (localizable) functors of Hilbertspaces into von Neumann algebras. Algebraic nets are supposed to cover the area *beyond* such localizable function algebras i.e. situations in which the locality principle is not implemented through functions but in a more noncommutative fashion. But in order to explicitly construct examples beyond CCR and CAR one has to start from the latter because these are the only nets which we thoroughly know.

6 Constructive Attempts for $d = 2 + 1$ Anyons

The most natural idea for the construction of "free" fields with braid group statistics is the adaption of Wigner's representation theory of the Poincaré group to this problem. Since the "little group" is the abelian $U(1)$, the Wigner wave function are one-component functions on the mass hyperboloid which transform under Lorentz-transformations as

$$(U(\Lambda)\psi)(p) = e^{is\phi_W(p,\Lambda)}\psi(\Lambda^{-1}p) \tag{24}$$

where the Wigner phase $\phi_W(p, \Lambda)$ is a computable real function which, as a result of $\mathbf{R} \subset \widetilde{SO}(2,1)$ represents an element of the covering of the Lorentz-group. In case of charged particles one has to double the wave-function space in order to obtain a representation of TCP.

In the next step the Wigner theory has to be extended by a localization concept which was not known to Wigner (i.e. different from the Newton-Wigner localization). One considers real subspaces $\mathcal{H}_R$ of $\mathcal{H}$, similar to those of chiral current algebra of the previous section. The relevant concept of causally disjoint localization is then defined in terms of the symplectic complement using the symplectic form:

$$\sigma(f,g) = Im(f,g)_{\mathcal{H}}, \qquad \mathcal{H} = \mathcal{H}_W \oplus \mathcal{H}_W^{anti} \tag{25}$$

Without local fields it is generally very difficult to explicitly find these local subspaces $\mathcal{H}_R(O)$ which are associated to regions (example: double cones) $\mathcal{O}$ in $d = 2+1$ Minkowski space.

However there is one exception: the (Rindler or Bisognano-Wichmann) "wedge" regions. For wedges the modular Tomita-Takesaki theory becomes geometric. On the Wigner space this theory admits a kind of "pre" (or "first-quantized")-version [38]. Instead of the modular conjugation J and the modular group Δ^{it} operators in Fockspace, we have operators j and δ^{it} acting geometrically on the Wigner wave function space. The pre-Tomita operator

$$s = j\delta^{1/2} \tag{26}$$

is an unbounded closable involutive ($s^2 = 1$) operator on $\mathcal{H}$ [38] which serves to define a real subspace $\mathcal{H}_R$ (wedge) as the (-1) eigenspace of s

$$\mathcal{H}_R(wedge) = \text{closure of real linear span } \{\psi \in \mathcal{H} \quad , \quad s\psi = -s\} \tag{27}$$

The s is explicitly known, since δ can be computed from the Lorentz-boost (which conserves the wedge) and j is obtained from the TCP operator ϑ on $\mathcal{H}$ by splitting off a π-rotation around the x-axis (in case of the standard $t - x$ wedge).

In the standard $d = 3+1$ Wigner theory, this idea was recently used [39] to define local nets in a direct fashion, i.e. without local fields as intermediaries. Since double cones can be obtained as intersection of wedges, the double cone algebras can be defined in terms of intersections of wedge algebras and the isotonic and causal structure of this so defined net follows without reference to local fields [39]. It is very interesting to extend these ideas to $d = 2+1$ anyonic spin representations [40]. In this case the statistical phase belonging to the Wigner spin $s \neq$ integer shows up as a mismatch between the j-transformed real subspace (the symplectic complement) and the geometric opposite wedge (obtained by a spatial rotation):

$$j \cdot \mathcal{H}_R(\text{wedge}) \neq \mathcal{H}_R(\text{opp.wedge}) \tag{28}$$

The operator which removes this mismatch is related to a Klein-transformation. In case of e.g. $\mathbf{Z}_N$-anyons this Klein-transformation T is again a suitable square root of the 2π-rotational in Theorem 4.

A significant difference between the (semi)integer spin case and genuine fractional spin appears, if one investigates[40] the possibility of having sharper localization than just wedge localization. It turns out that for the fractional case no compact localization is possible, i.e. certain intersection of $\mathcal{H}_R$ (wedge)-spaces are empty. In those cases one expects the semi-infinite spacelike string localization to be the best possibility. Therefore one looks for string-like localized fields.

The standard way to obtain fields from the Wigner wave functions is to "factorize" Wigner's wave functions into covariant wave functions times a generalization of the u and v spinors. Candidates for covariant wave functions in $d = 2+1$

were first proposed by Mund and Schrader [41] [23].

$$\psi_{cov}(p.g) := F(L^{-1}(p)g)\psi_w(p) \tag{29}$$

with F a (yet unspecified) kinematic function on the covering group $\widetilde{SO}(2,1)$ which has the following equivariance law with respect to the little group:

$$F(w\cdot g) = e^{isw}F(g) \quad , \quad w \in \mathbf{R} \subset S\widetilde{(2,1)} \tag{30}$$

Indeed this equivariance leads to the covariant transformation

$$(U(g)\psi_{cov.})(p,g') = \psi_{cov}(g^{-1}p, gg'). \tag{31}$$

Introducing momentum space creation and annihilation operator and covariant operators in x-space according to

$$\psi(x,g) = \int \frac{d^2p}{2w}(e^{ipx}a(p,g) + e^{-ipx}a^*(p,g)) \tag{32}$$

we have:

$$U(g')\psi(x,g)U^+(g') = \psi(\Lambda(g')x, g'g) \tag{33}$$

and

$$(\Omega, \psi(x_1,f_1)\psi(x_2,g_2)\Omega) = \int \frac{d^2p}{2w}e^{ip(x_1-x_2)}\bar{F}(L^{-1}(p)g_1)\cdot F(L^{-1}(p)g_2) \tag{34}$$

Choosing x_2 and g_2 "opposite" to x_1, g_1 i.e.

$$\begin{aligned} x_2 &= R(\pi)x_1,\ R(\pi) \text{ rotation by } \pi \\ g_2 &= R(\pi)g_1 \end{aligned} \tag{35}$$

one obtains the statistics factor in agreement with our previous wedge localization:

$$(\Omega, \psi(x_1,g_1)\psi(x_2,g_2)\Omega) = e^{2\pi is}(\Omega, \psi(x_2,g_2)\psi(x_1,g_1)\Omega). \tag{36}$$

In order to obtain fields which have a string-like localization, one has to specify appropriate functions F. As Fredenhagen and Gaberdiel [23] showed, the following specification leads to lightlike strings.

$$F(g) = e^{ig(0)s} \tag{37}$$

with $g(u) \subset \mathbf{R}$ for $u \in \mathbf{R} = \tilde{S}^1 \subset SO\widetilde{(2},1)$ and g acting in the natural way as a fractional $SO(2,1)$ transformation:

$$\begin{aligned} g &= (\gamma, w) \\ (\gamma,w)(u) &= w + arg\frac{e^{iu}+\gamma}{1+e^{-iu}\bar{\gamma}} \end{aligned} \tag{38}$$

These lightlike strings should be the singular limit of more natural space-like strings, but a detailed investigation of the letter is still missing.

A precise comparison with the results obtained on the basis of a Chern-Simons picture (i.e. a Chern-Simons vector potential interacting with spinor free matter) is difficult because the Chern Simons approach has not been perused far enough in order to obtain operators fulfilling braid group statistics. However on a very formal level it appears that the modified spinor matter fields are of the following type [42]

$$\psi =: e^{bil(\psi_0,\psi_0^+)}\psi_0 : \tag{39}$$

This is similar to the formula of the order parameter in the $d = 1 + 1$ massive Ising field theory. Such a structure would have a more complicated combinatorics than the previous "Wigner-anyons". Whereas the latter are *elementary* objects, the Chern-Simons anyons are *composite* represented in the Fermion Fockspace. I presently have no comment on this discrepancy, except the obvious remark that Chern-Simons anyons are perhaps not the simplest possible realization of $d = 2 + 1$ braid group statics.

The problem of constructing genuine $d = 2 + 1$ "free" plektons by algebraic methods is at least as complicated as that "anyonization" of conformal plektons of the previous section and there are presently no constructive ideas. This area of research promises to be interesting, lively and controversial for some time to come.

7 Concluding Remarks

In this work we argued that algebraic QFT presently plays its most constructive role in $d \leq 2 + 1$ QFT's with plektonic statistics were Lagrangian method fail.

For $d = 3 + 1$ QFT's the algebraic method suggests two areas of potential progress: a reinvestigation of perturbative problems in the sense of section 4, and a better understanding of the physical principles behind gauge theories.

Algebraic QFT always maintained a critical distance to an alleged "local gauge principle". Therefore there is no particular reason for an algebraic field theorist to react towards some new paradoxical looking situations (e.g. the alleged clash between the Seiberg-Witten duality and the "local gauge principle"[15]) with surprise. To the contrary, a critical evaluation from an algebraic (rather than a differential geometric) standpoint would lead one to believe that among all attempts to press a new physical reality into the quasiclassical straight-jacket of Lagrangian quantization, local gauge theories is the most useful one. The formal gauge idea suggests a new potentially fruitful concepts of algebraic QFT: the enhancement of degrees of freedom in asymptotic limit theories ("asymptotic freedom") [16].

Earlier attempts to "problematize" the notion of magnetic field in terms of operator 2-cohomology on the same level of depth as the problematization of the

notion of charge in the theory of superselection sectors (which can be reinterpreted as local operator 1-cohomology[39]) have not met with much success [39]. In the light of recent observations on the e.m. duality in supersymmetric QFT's, the main problem from the algebraic point of view is the understanding of a local physical property of the net generated by the field strength $F_{\mu\nu}$ which excludes the "free phase", i.e. excludes the simultaneous absence of electric as well as magnetic charges. The free electromagnetic field has, as we have seen, a peculiar obstruction, not shared by other massive free fields. It violates Haag duality for topologically nontrivial regions. It is believed that $F_{\mu v}$'s with nontrivial charges may not show this obstruction. For dual order-disorder situations in $d \leq 2+1$ theories such a criterion would not be necessary, since the case that both dual variables leave the vacuum sector invariant (i.e. do not create charges) is automatically excluded. But a classification of superselection rules coming from Maxwellian-like e. and m. charges remains difficult, since the bad infrared behaviour prevents a localization of charges in arbitrarily thin semi-infinite spacelike cones.

The lesser popularity of algebraic QFT as compared to the geometric quantization approaches seems to be a result of its conservative attitude with respect to inventions.

However underneath the conservative surface of the algebraic method hides a revolutionary new way of looking at QFT, a change of paradigm with respect to Lagrangian quantization. In more philosophical terms, the latter complies with the Newtonian way of understanding physical reality: a space-time manifold with a material content. The net point of view of algebraic QFT with its relations and inclusions between isomorphic local algebras (hyperfinite III_1 von Neumann factors) on the other hand harmonizes much more with Leibniz view of reality as coming about through the relations of monades ($\simeq$ the hyperfinite III_1 von Neumann factors representing the local algebras). This is of course also the underlying philosophy about mathematics in the Jones subfactor theory.

Looking at present QFT-related theoretical issues, it is not difficult to find a rough division into three topics: Renormalization group QFT, String theory and algebraic QFT.

The first one has led to a philosophy [44] that one should be happy with cutoff Lagrangians. The new aspect of the philosophy underlying the renormalization group in QFT is not that it states that reality is like an "infinite onion", each layer has its principles only to be superseded by the next one. Physics was always like this, and old principles were always eventually rendered limiting cases of new ones. Rather it resides in the message that to strife for conceptual and mathematical consistency in each layer is not required any more, since our lack of knowledge about the next layer can be summarily taken care of by cutoffs (even though this violates our *present* principles). With other words the principles with which this century had such a glorious start are an illusion, "effective" theories with cutoffs are the new goal. This ideology looks modest, apart from its tendency to

label "reductionist" viewpoints (as the one in this article) as scientific arrogance [44, page 157]. To me it appears as the physicists analogue of Fujikawas socio-economical ideology.

The second topic, namely string theory will have to make a big effort in order not to end as the "mathematically most useful physical phlogiston theory of the 20^{th} century" [53]. Rampant mathematics unbridled by physical principles and concepts does not seem to be the way out of the most profound crisis.

Areas close to ones own research one usually criticizes more mildly. However it is not a secret, that algebraic field theorist are a bit like religious preachers. Their everyday life (teaching courses on QFT by using a 40 year old formalism) is often quite apart from those high conceptual values they preach. My hope is that this will change in the near future. Algebraic QFT is attractive to me, because it remained a "family" activity and one does not feel compelled to make unfulfillable claims.

There is an interesting historical precedent that a conservative approach, i.e. one which is faithful to physical principles but revolutionary in their implementation may at the end be the more successful one. I am again referring to the discovery of renormalization theory in 1948/49 which did not use any of the many inventions of those days but just consisted in an elaboration of a conservative ideas and the pursuit of its intrinsic logic. For a very informative account I refer to Weinberg's new textbook [22].

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